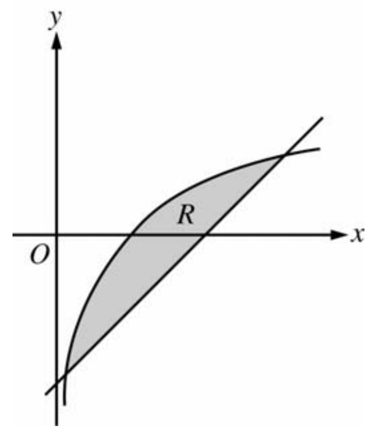
Question 1

Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

(c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.



$\ln(x) = x - 2$ when $x = 0.15859$ and 3.14619.
Let $S = 0.15859$ and $T = 3.14619$

Solution	Points
(a) Area of $R = \int_S^T (\ln(x) - (x - 2))\,dx = 1.949$	$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{answer} \end{cases}$
(b) Volume $= \pi \int_S^T \left((\ln(x) + 3)^2 - (x - 2 + 3)^2 \right) dx$ $= 34.198$ or 34.199	$3: \begin{cases} 2: \text{integrand} \\ 1: \text{limits, constant, and answer} \end{cases}$
(c) Volume $= \pi \int_{S-2}^{T-2} \left((y + 2)^2 - \left(e^y\right)^2 \right) dy$	$3: \begin{cases} 2: \text{integrand} \\ 1: \text{limits and constant} \end{cases}$

 1A1

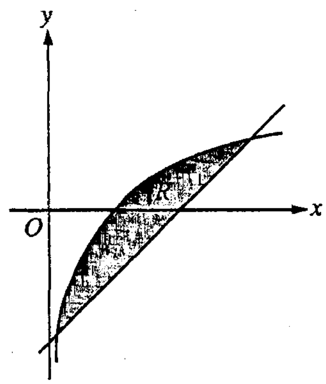
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$y = \ln x \qquad y = x - 2$

$\ln x = x - 2 \qquad x = .1586, \; x = 3.1462$

$$\int_{.1586}^{3.1462} (\ln x - x + 2)\, dx = \boxed{1.9491}$$

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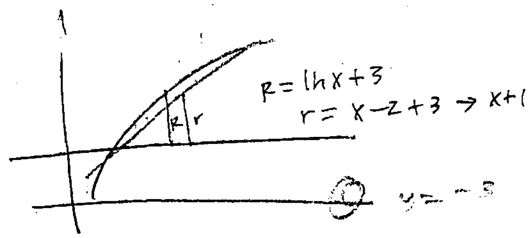
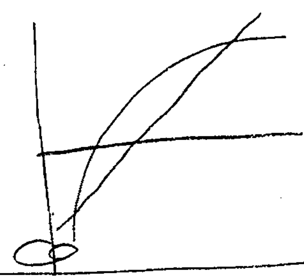
Continue problem 1 on page 5.

Work for problem 1(b)



$$\pi \int_{.1586}^{3.1462} \left[(\ln x + 3)^2 - (x - 2 + 3)^2 \right] dx = \boxed{34.1986}$$

Work for problem 1(c)



$$\boxed{2\pi \int_{-.1586}^{3.1462} \left[x(\ln x - x + 2) \right] dx}$$

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1B1

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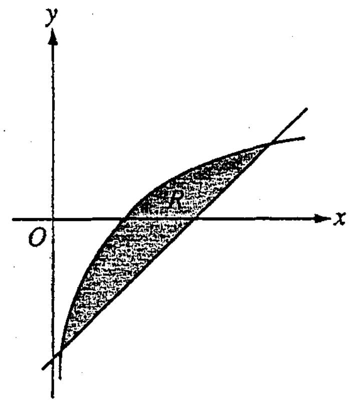
CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$y = \ln x \quad y = x - 2$

$\ln x = x - 2 \quad x = .158594 + 3.14619$

$A = \int_{.158594}^{3.14619} (\ln x - (x-2))\,dx$

$A = 1.949$

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Continue problem 1 on page 5.

1B2

Work for problem 1(b)

$V = \pi \int_{.158594}^{3.14619} ((-3-\ln x)^2 - (-3-(x-2))^2)\,dx$

$V = 10.886\pi$

$V = 34.199$

Work for problem 1(c)

$x-2=0$ $\ln x = 0$

$x=2$ $x=1$

$V = \pi \int_{1}^{3.14619} ((\ln x)^2 - (x-2)^2)$

$V = \pi \int_{.158594}^{2} ((x-2)^2 - (\ln x)^2)$

$V = \pi \int_{1}^{3.14619} ((\ln x)^2 - (x-2)^2) + \pi \int_{.158594}^{2} ((x-2)^2 - (\ln x)^2)\,dx$

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1C1

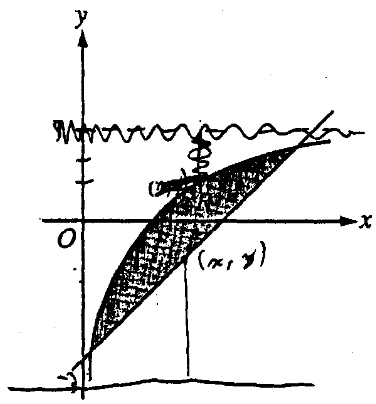
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$a = \cancel{\int_0^{3.138}(x-2)-\ln x}$ $\int_0^{3.138} \ln x - (x-2)\, dx$

$\boxed{a = 1.80}$

Continue problem 1 on page

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1C2

Work for problem 1(b)

$V = \cancel{\#}3 - y$

$V = 2\pi \int_0^{3.138} (3-(x-2))(\ln x - (x-2))\,dx$

$V = 2\pi \cdot 6.299$

$\boxed{V = 12.599\pi}$

Work for problem 1(c)

$V = 2\pi \int_0^{3.138} x(\ln x - (x-2))\,dx$

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GO ON TO THE NEXT PAGE.

Question 1

Overview

This problem gave two graphs that intersect at $x = 0.15859$ and $x = 3.14619$. A graphing calculator was required to find these two intersection values. Students needed to use integration to find an area and two volumes. In part (a) students had to find the area of the region bounded by the two graphs. In part (b) students had to calculate the volume of the solid generated by rotating the region about the horizontal line $y = -3$, a line that lies below the given region. Part (c) tested the students' ability to set up an integral for the volume of a solid generated by rotating the given region around a vertical axis, in this case the y-axis. The given functions could be solved for x in terms of y, leading to the use of horizontal cross sections in the shape of washers and an integral in terms of the variable y. Although no longer included in the *AP Calculus Course Description*, the method of cylindrical shells could also be used to write an integral expression for the volume in terms of the variable x.

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

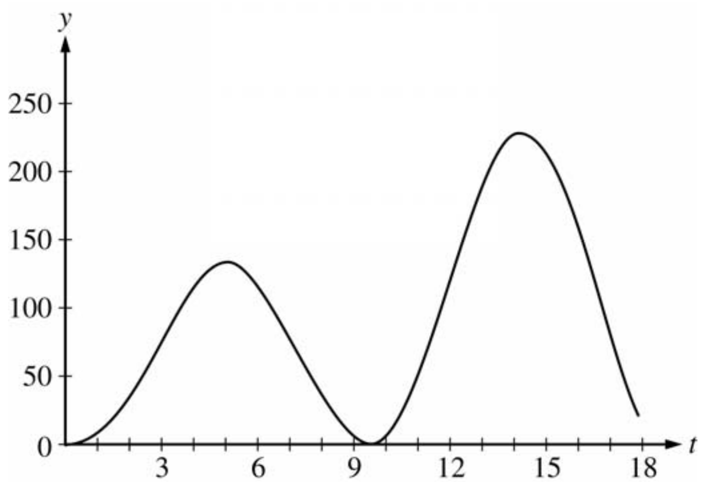
The student earned 6 points: 3 points in part (a) and 3 points in part (b). In part (a) the student has the correct integrand, which earned the first point. The definite integral has the correct limits to three decimal places, which earned the second point. The answer is correct to three decimal places and earned the third point. In part (b) the student has the correct integrand, which earned the first 2 points. The extra factor of -1 in each integrand term does not cause a problem since it is an equivalent form to the standard. The student correctly evaluates the integral and produces the correct answer to three decimal places. In part (c) the student does have a difference of squares, which provides entry into the problem, but since neither term is correct the student did not earn either integrand point. The student was not eligible for the limits/constant point since no integrand point was earned.

Sample: 1C
Score: 3

The student earned 3 points: 1 point in part (a) and 2 points in part (c). In part (a) the student has the correct integrand, which earned the first point. The limit point was not awarded because the student's limits are not correct to three decimal places. Since the lower limit of 0 is not within the acceptable range for limits, the student was not eligible for the answer point. In part (b) the student attempts a cylindrical shell setup, but the integrand is incorrect, so neither point was earned. Since neither integrand point was earned, the student was not eligible for the limits/constant/answer point. In part (c) the student correctly provides the integrand for the cylindrical shells method and earned the first 2 points. The student's limits, in particular the lower value of 0, are not in the acceptable range, so the limits/constant point was not earned.

Question 2

At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $y = L(t)$ is shown above.



(a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

(b) Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

(c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

(a) $\int_0^{18} L(t)\,dt \approx 1658$ cars

$2: \begin{cases} 1: \text{setup} \\ 1: \text{answer} \end{cases}$

(b) $L(t) = 150$ when $t = 12.42831,\ 16.12166$
Let $R = 12.42831$ and $S = 16.12166$
$L(t) \ge 150$ for t in the interval $[R, S]$
$\frac{1}{S - R}\int_R^S L(t)\,dt = 199.426$ cars per hour

$3: \begin{cases} 1: t\text{-interval when } L(t) \ge 150 \\ 1: \text{average value integral} \\ 1: \text{answer with units} \end{cases}$

(c) For the product to exceed 200,000, the number of cars turning left in a two-hour interval must be greater than 400.

$\int_{13}^{15} L(t)\,dt = 431.931 > 400$

$4: \begin{cases} 1: \text{considers 400 cars} \\ 1: \text{valid interval } [h, h+2] \\ 1: \text{value of } \int_h^{h+2} L(t)\,dt \\ 1: \text{answer and explanation} \end{cases}$

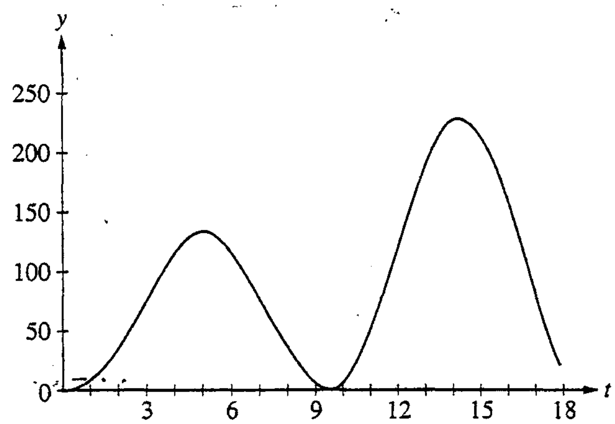
OR

The number of cars turning left will be greater than 400 on a two-hour interval if $L(t) \ge 200$ on that interval.
$L(t) \ge 200$ on any two-hour subinterval of $[13.25304, 15.32386]$.

OR

$4: \begin{cases} 1: \text{considers 200 cars per hour} \\ 1: \text{solves } L(t) \ge 200 \\ 1: \text{discusses 2 hour interval} \\ 1: \text{answer and explanation} \end{cases}$

Yes, a traffic signal is required.



Work for problem 2(a)

$L(t) = 60\sqrt{t}\,\sin^2\left(\frac{t}{3}\right)$

$\int_0^{18} 60\sqrt{t}\,\sin^2\left(\frac{t}{3}\right)dt$

$= 1657.8237$

$= 1658$ cars

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Continue problem 2 on page 7.

Work for problem 2(b)

$L(t) = 150 = 60\sqrt{t}\,\sin^2\left(\frac{t}{3}\right)$

$60\sqrt{t}\sin^2\frac{t}{3} - 150 = 0$

$t = 12.42831$ or 16.121657

$\frac{1}{b-a}\int_a^b L(t) = \frac{1}{16.121657-12.42831}\int_{12.42831}^{16.121657} 60\sqrt{t}\sin^2\frac{t}{3}\,dt$

$= \frac{1}{3.693347}\left[736.54986\right] = 199.4261195$

$12.428 \le t \le 16.121657$

199.426 cars/hr

Work for problem 2(c)

Cars turning left × oncoming cars going straight

(Cars turning left) 500 ≤ 200,000

cars turning left = 400

$\int_{14}^{16} L(t) = \int_{14}^{16} 60\sqrt{t}\,\sin^2\left(\frac{t}{3}\right)dt = 412.26$

Yes, there will need to be a signal because between the interval t=14 and t=16, 412 cars turn left. When you multiply that by 500, it exceeds 200,000

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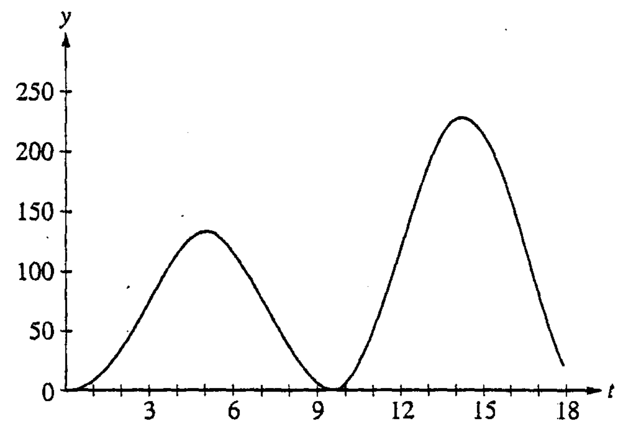

Work for problem 2(a)

$\int_0^{18} L(t)\,dt$

$\int_0^{18} \left[60\sqrt{t}\sin^2(t/3)\right] dt$

1658

1658 total cars turn left through the intersection between the hours of 0 hours and 18 hours.

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Continue problem 2 on page 7.

Work for problem 2(b)

$L(t) > 150$

$60\sqrt{t}\sin^2(t/3) > 150$

$t = [12.42831, 16.121657]$

L(t) is greater than 150 at all hours between 12.42831 hours and 16.121657 hours.

average value:

$\frac{1}{b-a}\int_a^b L(t)\,dt$

$\frac{1}{16.121657 - 12.42831}\int_{12.42831}^{16.121657} [60\sqrt{t}\sin^2(t/3)]\,dt$

199.426

The average number of cars turning left between 12.42831 hours and 16.121657 hours is 199.426 cars.

Work for problem 2(c)

500 straight cars/2 hours

200,000 = straight cars * left cars
(500)

$\frac{200,000}{500} = 400$

Therefore, 400 left turn cars during a 2-hour interval would require a trafficsignal.

Yes, the intersection does require a traffic signal. At 13.253 hours, L(t) = 200 and increases upward. At at least 200 cars per hour, the product would exceed 200,000 therefore requiring a signal. The flow of cars does not drop below 200 cars/hr until 15.323 hours.

$\int_a^{a+2} L(t)\,dt \geq 400?$

13.253, 15.323

$L(t) \geq 200?$

$L(t) \geq 200$ at

$[13.253, 15.323]$

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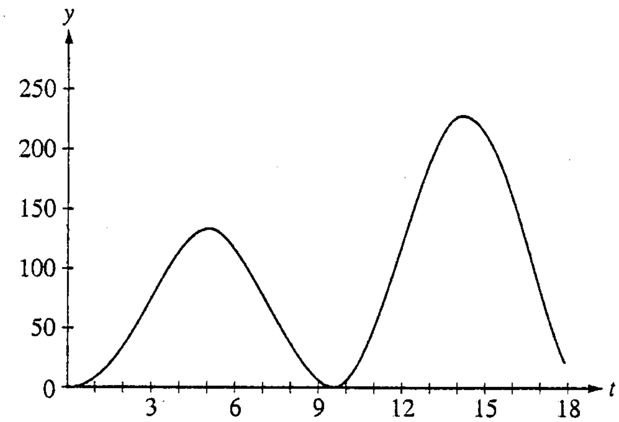

Work for problem 2(a)

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total cars $\Rightarrow \int L(t)$

$= \int_0^{18} 60\sqrt{t}\,\sin^2\left(\frac{t}{3}\right)$

≈ 1658 cars

Continue problem 2 on page 7.

Work for problem 2(b)

$L(t) \geq 150$ from ≈ 12.428 to ≈ 16.120

$\frac{1}{b-a}\int_{12.428}^{16.120} 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$

$\frac{1}{16.120-12.428}(736.34771)$

$\frac{1}{3.692}(736.34771) \approx 199.4$ = avg. # of cars turning left

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Work for problem 2(c)

(total # of cars turning left)(500 straight cars) $\overset{?}{\geq}$ 200,000

$\left(\int_{12.428}^{16.120} 60\sqrt{t}\sin^2\left(\frac{t}{2}\right)\right)(500)$

$(736.34771)(500) = 368173.855$

$368174 \geq 200.000$

$\boxed{\text{Yes}}$

The product of cars turning (L) and cars going straight is greater than 200,000, and requires the installation of a traffic signal.

GO ON TO THE NEXT PAGE.

Question 2

Overview

This problem gave students a function that modeled the rate, in cars per hour, at which cars turn left at a given intersection. In part (a) students had to use the definite integral to find the total number of cars that turned left in a given time period. In part (b) students had to use their graphing calculators to find the time interval during which the rate equaled or exceeded 150 cars per hour and then compute the average value of the rate over this time interval. Part (c) described a condition that would require the installation of a traffic signal at an intersection and asked students to decide if a signal was necessary at this particular intersection. Students could do this in several ways. For example, students could recognize that a signal would be required if the number of cars that turn left over a two-hour time interval exceeds 400 cars and then find an appropriate interval. Or students might recognize that a signal would be required if the average value of the rate at which cars turn left over any two-hour time interval exceeds 200 cars per hour. Since the rate itself exceeds 200 cars per hour during the interval $13.253 < t < 15.324,$ the average value of the rate will also exceed 200 cars per hour during this time interval of length greater than two hours, and thus a signal would be required.

Sample: 2A
Score: 9

The student earned all 9 points. In part (c) all 4 points were earned for a correct argument based on the value of the integral of $L(t)$ over a two-hour period.

Sample: 2B
Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), and 2 points in part (c). In part (a) the student provides the correct definite integral and its correct evaluation to the nearest whole number, which earned both points. In part (b) the student determines the correct interval, which earned the first point, and provides the correct average value setup to earn the second point. The third point was not earned because the units given for the computed average value are not correct. In part (c) the student earned the first point for observing that 400 left turns in a two-hour interval are needed. The second point was earned for correctly identifying the interval on which $L(t) \geq 200.$ The student never compares the length of the given interval to 2 and thus did not earn the third point. The student was not eligible for the fourth point.

Sample: 2C
Score: 4

The student earned 4 points: 2 points in part (a), 1 point in part (b), and 1 point in part (c). In part (a) the student provides the correct definite integral and its correct evaluation to the nearest whole number, which earned both points. In part (b) the upper limit for the student's interval is not correct in the third decimal place, so the first point was not earned. The second point was earned for the average value setup since the student provides the correct value for the given integral, divided by the length of the interval. The third point was not earned because the student rounds the answer to the first decimal place instead of the required three. In part (c) the first point was earned for the observation that the product of the total number of cars turning left and 500 needs to be greater than 200,000. The student does not consider a two-hour interval, so no further points were earned.

Question 3

An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \sin^{-1}\left(1 - 2e^{-t}\right) \text{ and } \frac{dy}{dt} = \frac{4t}{1 + t^3}$$

for $t \ge 0$. At time $t = 2$, the object is at the point $(6, -3)$. (Note: $\sin^{-1} x = \arcsin x$)

(a) Find the acceleration vector and the speed of the object at time $t = 2$.

(b) The curve has a vertical tangent line at one point. At what time t is the object at this point?

(c) Let $m(t)$ denote the slope of the line tangent to the curve at the point $(x(t), y(t))$. Write an expression for $m(t)$ in terms of t and use it to evaluate $\lim_{t \to \infty} m(t)$.

(d) The graph of the curve has a horizontal asymptote $y = c$. Write, but do not evaluate, an expression involving an improper integral that represents this value c.

(a) $a(2) = \langle 0.395 \text{ or } 0.396, -0.741 \text{ or } -0.740 \rangle$

Speed $= \sqrt{x'(2)^2 + y'(2)^2} = 1.207$ or 1.208

$2: \begin{cases} 1: \text{acceleration} \\ 1: \text{speed} \end{cases}$

(b) $\sin^{-1}\left(1 - 2e^{-t}\right) = 0$

$1 - 2e^{-t} = 0$

$t = \ln 2 = 0.693$ and $\frac{dy}{dt} \ne 0$ when $t = \ln 2$

$2: \begin{cases} 1: x'(t) = 0 \\ 1: \text{answer} \end{cases}$

(c) $m(t) = \frac{4t}{1 + t^3} \cdot \frac{1}{\sin^{-1}\left(1 - 2e^{-t}\right)}$

$$\lim_{t \to \infty} m(t) = \lim_{t \to \infty} \left(\frac{4t}{1 + t^3} \cdot \frac{1}{\sin^{-1}\left(1 - 2e^{-t}\right)} \right) = 0\left(\frac{1}{\sin^{-1}(1)} \right) = 0$$

$2: \begin{cases} 1: m(t) \\ 1: \text{limit value} \end{cases}$

(d) Since $\lim_{t \to \infty} x(t) = \infty$,

$$c = \lim_{t \to \infty} y(t) = -3 + \int_2^{\infty} \frac{4t}{1 + t^3}\, dt$$

$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{initial value consistent} \\ \quad \text{with lower limit} \end{cases}$

Work for problem 3(a)

speed: $\frac{dx}{dt} = \arcsin(1 - 2e^{-2}) = .817$

$\frac{dy}{dt} = \frac{4(2)}{1+2^3} = \frac{8}{9}$

$v = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$v = 1.208$

accel. vector: $\frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{1}{\sqrt{e^t - 1}}$

$\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{-4(2t^3-1)}{(t^3+1)^2}$

$t=2$

$\langle .396, -.741 \rangle$

Work for problem 3(b)

vert tang: $\frac{dy}{dx}$ when $dx = 0$, $t = ?$

$0 = \arcsin(1 - 2e^{-t})$

$t = .693$

Continue problem 3 on page 9.

Work for problem 3(c)

$$m(t) = \frac{4t}{(1+t^3)\arcsin(1-2e^{-t})}$$

$$\lim_{t\to\infty} \frac{4t}{(1+t^3)\arcsin(1-2e^{-t})} = 0$$

Work for problem 3(d)

$$\int_2^\infty \frac{4t}{1+t^3}\,dt - 3$$

END OF PART A OF SECTION II
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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Work for problem 3(a)

acceleration vector = (.396, -.741)

Speed = 1.208

Work for problem 3(b)

Verticle tangent where $\frac{dx}{dt} = 0$

$\frac{dx}{dt} = 0$ at $t = .693$

Continue problem 3 on page 9.

Work for problem 3(c)

$$m(t) = \frac{4t}{(1+t^3)\sin^{-1}(1-2e^{-t})}$$

$$\lim_{t \to \infty} m(t) = 1$$

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Work for problem 3(d)

$$\int_1^\infty \frac{4t}{(1+t^3)\sin^{-1}(1-2e^{-t})}\,dt$$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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Work for problem 3(a)

$v_x = \sin^{-1}(1-2e^{-t})$

$v_x(2) = .817$

$a_x = \dfrac{1-2e^{-t}}{\sqrt{1-(1-2e^{-t})^2}}$

$a_x(2) = 1.066$

$v_y = \dfrac{4t}{1+t^3}$ $\dfrac{1}{(1+t^3)^{-1}}$

$v_y(t) = .889$

$v = \langle .817, .889 \rangle$

$a_y = (4t \cdot -1(1+t^3)^{-2} \cdot 3t^2) + (4 \cdot \frac{1}{1+t^3})$

$a_y(2) = -.741$

$a = \langle 1.066, -.741 \rangle$

Work for problem 3(b)

$t = 1$

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Continue problem 3 on page 9.

Work for problem 3(c)

$$m(t) = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dx}$$

$$m(t) = \frac{\frac{4t}{1+t^3}}{\sin^{-1}(1-2e^{-t})}$$

$$\lim_{t\to\infty} m(t) = 0$$

Work for problem 3(d)

$$\int_0^{\infty} \frac{\frac{4t}{1+t^3}}{\sin^{-1}(1-2e^{-t})}$$

$$= \lim_{b\to\infty}\int_0^b \frac{\frac{4t}{1+t^3}}{\sin^{-1}(1-2e^{-t})}\,dt$$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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Question 3

Overview

This problem dealt with particle motion in the plane. Students were given the rate of change of the x- and y-coordinates as functions of time and the initial position of a particle at time $t = 2.$ Part (a) asked for the acceleration vector and speed of the object at time $t = 2.$ Parts (b) and (c) dealt with lines tangent to the curve along which the object moves. In part (b) students had to find the time at which the curve has a vertical tangent line, and in part (c) students had to find a general expression for the slope of the line tangent to the curve at an arbitrary point on the curve. Students were also asked to evaluate the limit of this slope as $t \to \infty$. Part (d) tested the students' ability to use the Fundamental Theorem of Calculus to write an improper integral that represented the value of the horizontal asymptote for the graph of the curve.

Sample: 3A
Score: 9

The student earned all 9 points. Note that because this is a calculator-active question, the student was not required to show work leading to the answers presented in parts (a), (b), and (c). In particular, analytic expressions for the second derivatives were not required to earn the point for the acceleration vector in part (a).

Sample: 3B
Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), 1 point in part (c), and 1 point in part (d). In part (a) the student correctly presents an acceleration vector and a value for speed at time $t = 2$ to three decimal places. The student earned both points for part (a). In part (b) the student sets $x'(t) = 0$ and earned the first point. The student goes on to solve correctly for t to three decimal places and earned the second point. In part (c) the student presents a correct expression for $m(t)$ in terms of t and earned the first point. The limit is incorrectly evaluated, and the student did not earn the second point. In part (d) the student presents $m(t)$ as the integrand and did not earn the integrand point. The student presents limits of integration of the form $a \geq 0$ for the lower limit and infinity for the upper limit and earned the limits point. The student does not consider an initial condition and therefore did not earn the third point.

Sample: 3C
Score: 3

The student earned 3 points: 2 points in part (c) and 1 point in part (d). In part (a) the student presents analytic expressions for the x- and y-components of the acceleration vector. The incorrect expression in the numerator of the x-component of the acceleration vector leads to an incorrect numerical value in the presentation of the acceleration vector. The student did not earn the first point. The velocity vector at $t = 2$ is given but the speed is not calculated. The student did not earn the second point. In part (b) no equation is given and an incorrect t value is presented. The student did not earn either point. In part (c) the student gives a correct expression for $m(t)$ in terms of t and earned the first point. The limit is then correctly evaluated, and the student earned the second point. Note that the student was not required to show justification to earn the limit evaluation point. In part (d) the student presents an integral with the incorrect integrand and did not earn the first point. The student presents

Question 3 (continued)

limits of integration of the form $a \geq 0$ for the lower limit and infinity for the upper limit and earned the second point. Note that the use of the limit notation was not required to earn the second point. The student does not consider an initial condition and therefore did not earn the third point.

Question 4

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

(a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

(c) Rocket B is launched upward with an acceleration of $a(t) = \dfrac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

Solution	Scoring
(a) Average acceleration of rocket A is $\dfrac{v(80)-v(0)}{80-0} = \dfrac{49-5}{80} = \dfrac{11}{20}\ \text{ft}/\text{sec}^2$	1 : answer
(b) Since the velocity is positive, $\int_{10}^{70} v(t)\,dt$ represents the distance, in feet, traveled by rocket A from $t = 10$ seconds to $t = 70$ seconds. A midpoint Riemann sum is $20[v(20)+v(40)+v(60)]$ $= 20[22+35+44] = 2020$ ft	3 : $\begin{cases} 1 : \text{explanation} \\ 1 : \text{uses } v(20),\ v(40),\ v(60) \\ 1 : \text{value} \end{cases}$
(c) Let $v_B(t)$ be the velocity of rocket B at time t. $v_B(t) = \int \dfrac{3}{\sqrt{t+1}}\,dt = 6\sqrt{t+1} + C$ $2 = v_B(0) = 6 + C$ $v_B(t) = 6\sqrt{t+1} - 4$ $v_B(80) = 50 > 49 = v(80)$ Rocket B is traveling faster at time $t = 80$ seconds.	4 : $\begin{cases} 1 : 6\sqrt{t+1} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{finds } v_B(80)\text{, compares to } v(80)\text{,} \\ \quad\ \text{and draws a conclusion} \end{cases}$
Units of ft/sec^2 in (a) and ft in (b)	1 : units in (a) and (b)

NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Work for problem 4(a)

$$\text{avgval}_{a(t)} = \frac{v(b)-v(a)}{b-a}$$

$$\text{avgval}_{a(t)} = \frac{v(80)-v(0)}{80-0}$$

$$\text{avgval}_{a(t)} = \frac{49-5}{80-0}$$

$$\boxed{\text{avgval}_{a(t)} = 11/20 \text{ ft/s}^2}$$

Work for problem 4(b)

$\int_{10}^{70} v(t)\,dt$ gives the distance the rocket has traveled from time 10 seconds to 70 seconds in feet.

$$\Delta x = \frac{70-10}{3} = 20$$

$$p(t) = 20\left[v(20) + v(40) + v(60)\right]$$

$$p(t) = 20\left[22 + 35 + 44\right] = 20\left[101\right]$$

$$\boxed{p(t) = 2020 \text{ feet}}$$

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Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(c)

Rocket B

$a(t) = \frac{3}{\sqrt{t+1}}$

$v(t) = \int \frac{3}{\sqrt{t+1}}\,dt$

let $u = t+1$

$\frac{du}{dx} = 1$

$du = dx$

$v(t) = 3\int u^{-1/2}\,du$

$v(t) = \frac{3u^{1/2}}{1/2} + c$

$v(t) = 6(t+1)^{1/2} + c$

$2 = 6(0+1)^{1/2} + c$

$c = -4$

$v(t) = 6(t+1)^{1/2} - 4$

$v(80) = 6(80+1)^{1/2} - 4$

$v(80) = 6\sqrt{81} - 4$

$v(80) = 54 - 4$

$\boxed{v(80) = 50 \text{ ft/sec}}$

Rocket A

$\boxed{v(80) = 49 \text{ ft/sec}}$

Rocket B is traveling faster at t = 80 sec. Rocket B's velocity was found by $v(t) = \int a(t)\,dt$ and is 50 ft/sec. Rocket A's velocity was 49 ft/sec

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GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

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Work for problem 4(a)

$$\frac{1}{80}\left(9+8+7+6+5+4+3+2\right)=$$

$$=\frac{46}{80}=\frac{23}{40}\ \text{ft}/\text{sec}^2$$

Work for problem 4(b)

$\int_{10}^{70} v(t)\,dt$ indicates the distance the rocket traveled over the interval $10 \le t \le 70$. In this case, it indicates total distance because the rocket's velocity is only increasing during this time period.

$$\begin{array}{r} 70 \\ 22 \\ -44 \\ \hline 136 \end{array}$$

$(22+2(35)+44)=136$ ft.

Continue problem 4 on page 11.

Work for problem 4(c)

$3(t+1)^{-1/2}$

$v(t)=6\sqrt{t+1}+C$

$2=6\sqrt{0+1}+c$

$2=6+c$

$-4=c$

$v(t)=6\sqrt{t+1}-4$

$v(80)=6\sqrt{80+1}-4$

$6\cdot 9-4$

$v(80)=52$ ft/sec

Rocket B is traveling faster at t=80. The table states that Rocket A is traveling at 49 ft/sec. By taking the antideravitive for a(t) of Rocket B and solving with initial conditions for C, then substituting t=80, we find v(80)=52 ft/sec, faster than Rocket A.

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NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Work for problem 4(a)

$\text{acceleration}_{\text{ave}} = \frac{11}{20}$ feet/second^2

$49 - 5 = 44$

$\frac{44}{80}$

$\frac{11}{20}$

Work for problem 4(b)

The integral of the velocity is the position. $\int_{10}^{70} v(t)\,dt$ means the position of the rocket from 10 sec to 70 sec.

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Continue problem 4 on page 11.

Work for problem 4(c)

$a(t) = \frac{3}{\sqrt{t+1}}$

$t+1 = u$

$1\,dt = du$

$\int \frac{3}{\sqrt{t+1}}\,dt$

$= \int \frac{3\,du}{\sqrt{u}}$

$= \int 3u^{-\frac{1}{2}}\,du$

$= 3 \cdot 2u^{\frac{1}{2}}$

$= 6u^{\frac{1}{2}}$

$v(t) = 6\sqrt{t+1}$

$v(80) = 6\sqrt{80+1}$

$= 6\sqrt{81}$

$= 6 \times 9$

$= 54$ feet/second.

Rocket B is traveling faster at time = 80 sec.

The antiderivative of the acceleration gives the velocity. Using this, the velocity of Rocket B was discovered to be 54 feet per second at time = 80 seconds. Compared to the velocity of Rocket A, which is 49 feet per second, Rocket B is traveling faster.

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Question 4

Overview

This problem presented students with a table of velocity values for rocket A at selected times. In part (a) students needed to recognize the connection between the average acceleration of the rocket over the given time interval and the average rate of change of the velocity over this interval. In part (b) students had to recognize the definite integral as the total change, in feet, in rocket A's position from time $t = 10$ seconds to time $t = 70$ seconds and then approximate the value of this definite integral using a midpoint Riemann sum and the data in the table. Units of measure were important in both parts (a) and (b). Part (c) introduced a second rocket and gave its acceleration in symbolic form. The students were asked to compare the velocities of the two rockets at time $t = 80$ seconds. The velocity of rocket B could be determined by finding the antiderivative of the acceleration and using the initial condition or by using the Fundamental Theorem of Calculus and computing a definite integral.

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

The student earned 6 points: 2 points in part (b), 3 points in part (c), and the units point. The first line in part (a) is a correct but uncommon method. An error was made in the addition. In part (b) the explanation is acceptable; the student correctly identifies the midpoint values but the method is incorrect. In part (c) the fourth point was not earned due to an error in the computation of $v(80)$ for rocket B.

Sample: 4C
Score: 3

The student earned 3 points: 1 point in part (a) and 2 points in part (c). The units are correct in part (a) but missing in part (b). For the explanation in part (b), the integral does not represent the position but rather the displacement over the time interval $[10, 70]$. The constant of integration does not appear in part (c) so the student earned the antiderivative and comparison points only.

Question 5

Consider the differential equation $\frac{dy}{dx} = 5x^2 - \frac{6}{y-2}$ for $y \neq 2$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = -4$.

(a) Evaluate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $(-1, -4)$.

(b) Is it possible for the x-axis to be tangent to the graph of f at some point? Explain why or why not.

(c) Find the second-degree Taylor polynomial for f about $x = -1$.

(d) Use Euler's method, starting at $x = -1$ with two steps of equal size, to approximate $f(0)$. Show the work that leads to your answer.

Solution	Points
(a) $\left.\frac{dy}{dx}\right\vert_{(-1,-4)} = 6$ $\frac{d^2y}{dx^2} = 10x + 6(y-2)^{-2}\frac{dy}{dx}$ $\left.\frac{d^2y}{dx^2}\right\vert_{(-1,-4)} = -10 + 6\frac{1}{(-6)^2}6 = -9$	$3: \begin{cases} 1: \left.\frac{dy}{dx}\right\vert_{(-1,-4)} \\ 1: \frac{d^2y}{dx^2} \\ 1: \left.\frac{d^2y}{dx^2}\right\vert_{(-1,-4)} \end{cases}$
(b) The x-axis will be tangent to the graph of f if $\left.\frac{dy}{dx}\right\vert_{(k,0)} = 0$. The x-axis will never be tangent to the graph of f because $\left.\frac{dy}{dx}\right\vert_{(k,0)} = 5k^2 + 3 > 0$ for all k.	$2: \begin{cases} 1: \frac{dy}{dx} = 0 \text{ and } y = 0 \\ 1: \text{answer and explanation} \end{cases}$
(c) $P(x) = -4 + 6(x+1) - \frac{9}{2}(x+1)^2$	$2: \begin{cases} 1: \text{quadratic and centered at } x = -1 \\ 1: \text{coefficients} \end{cases}$
(d) $f(-1) = -4$ $f\left(-\frac{1}{2}\right) \approx -4 + \frac{1}{2}(6) = -1$ $f(0) \approx -1 + \frac{1}{2}\left(\frac{5}{4} + 2\right) = \frac{5}{8}$	$2: \begin{cases} 1: \text{Euler's method with 2 steps} \\ 1: \text{Euler's approximation to } f(0) \end{cases}$

NO CALCULATOR ALLOWED

5A

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Work for problem 5(a)

$\frac{dy}{dx} = 5x^2 - \frac{6}{y-2}\Big|_{(-1,-4)} = 5(-1)^2 - \frac{6}{-4-2} = 5 - \frac{6}{-6} = 6$

$\frac{d^2y}{dx^2} = 10x + \frac{6}{(y-2)^2}\frac{dy}{dx}$

$= 10x + \frac{6}{(y-2)^2}\cdot\underbrace{\left(5x^2 - \frac{6}{y-2}\right)}_{=6}$

$= 10x + \frac{36}{(y-2)^2}\Big|_{(-1,-4)} = -10 + \frac{36}{(-6)^2} = -10 + 1 = -9$

$\boxed{\frac{dy}{dx} = 6 \quad , \quad \frac{d^2y}{dx^2} = -9}$

Work for problem 5(b)

to be tangent, $\frac{dy}{dx}$ would equal zero, and y would also be zero

$\frac{dy}{dx} = 0 = 5x^2 - \frac{6}{y-2}$

$\frac{6}{y-2} = 5x^2$

$\frac{6}{-2} = 5x^2$

$x^2 = -\frac{3}{5}$

since x^2 cannot be negative, there is no point possible such that the x-axis is tangent to the graph.

Continue problem 5 on page 13.

Work for problem 5(c)

$P(x) = a_0 + a_1(x+1) + a_2(x+1)^2$

$P(-1) = a_0$

$P'(x) = a_1 + 2a_2(x+1)$

$P'(-1) = a_1$

$P''(x) = 2a_2$

$P''(-1) = 2a_2$

$f(x) = \sim$

$f(-1) = -4 \qquad a_0 = -4$

$f'(x) = 5x^2 - \frac{6}{y-2}$

$f'(-1) = 6 \qquad a_1 = 6$

$f''(x) = 10x + \frac{6}{(y-2)^2}\left[5x^2 - \frac{6}{y-2}\right]$

$f''(-1) = -9 \qquad 2a_2 = -9$

$a_2 = -4.5$

$\boxed{P(x) = -4 + 6(x+1) - 4.5(x+1)^2}$

Work for problem 5(d)

$y_1 = y_0 + f'(x_0, y_0)\Delta x \qquad (-1, -4)$

$y_1 = -4 + (5(-1)^2 - \frac{6}{-6})(0.5)$

$= -4 + (5 + 1)(\frac{1}{2})$

$= -4 + 3 = -1$

$\boxed{f(0) \approx \frac{5}{8}}$

$y_2 = y_1 + f'(x_1, y_1)\Delta x \qquad (-\frac{1}{2}, -1)$

$= -1 + (5(\frac{1}{4}) - \frac{6}{-3})(\frac{1}{2})$

$= -1 + (\frac{5}{4} + 2)\frac{1}{2}$

$= -1 + \frac{13}{8} = \frac{5}{8}$

NO CALCULATOR ALLOWED

5B

Work for problem 5(a)

$$\frac{dy}{dx}\Big|_{(-1,-4)} = 5(-1)^2 - \frac{6}{-4-2} = 5+1=6$$

$$\frac{d^2y}{dx^2} = 10x - \left[\frac{-6\frac{dy}{dx}}{(y-2)^2}\right] = 10x + \frac{6(5x^2-\frac{6}{y-2})}{(y-2)^2}$$

$$\frac{d^2y}{dx^2}\Big|_{(-1,-4)} = -10 + \frac{6(5+1)}{(-6)^2} = -10+\frac{36}{36} = -9$$

Work for problem 5(b)

$$0 = \frac{dy}{dx}$$

$$5x^2 - \frac{6}{y-2} = 0$$

$$5x^2 = \frac{6}{y-2}$$

$$5(-1)^2 = \frac{6}{-4-2}$$

$$5 \neq -1.$$

The graph can never be tangent to the x-axis because ~~the~~ that would mean that the slope was 0. The slope of f(x) can never equal 0.

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Continue problem 5 on page 13.

5B$_2$

Work for problem 5(c)

$f(-1) = -4$

$f'(-1) = 6$

$f''(-1) = -9$

$$f(x) = -4 + 6x - \frac{9x^2}{2!}$$

Work for problem 5(d)

x	dx	$\frac{dy}{dx}$	y	dy
-1	0.5	6	-4	3
-0.5		$\frac{13}{4}$	-1	$\frac{13}{8}$
0			$\frac{5}{8}$	

$$\left.\frac{dy}{dx}\right|_{(-.5,-1)} = \frac{5}{4} - \frac{6}{-3} = \frac{5}{4} + 2 = \frac{13}{4}$$

$f(0) \approx \frac{5}{8}$

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NO CALCULATOR ALLOWED

5C

Work for problem 5(a)

$\frac{dy}{dx} = 5x^2 - \frac{6}{y-2}$; $y=2$ $y=f(x)$

$f(-1) = -4$

$\frac{dy}{dx} = 5(-1)^2 - \frac{6}{-4-2}$

$= 5 - \frac{6}{-6}$

$= 6$

$\frac{d^2y}{dx^2} = 10x + \frac{6}{(y-2)^2} \frac{dy}{dx}$

$= 10x + \frac{6}{(y-2)^2}\left[5x^2 - \frac{6}{y-2}\right]$

$= 10(-1) + \frac{6}{(-4-2)^2}[6] \Rightarrow -10 + \frac{6}{36}\cdot 6 \Rightarrow -10 + \frac{1}{6}\cdot 6 = 9$

Work for problem 5(b)

$0 = 5x^2 - \frac{6}{y-2}$

$\frac{6}{y-2} = 5x^2$

Yes, it's possible for the x-axis to be tangent to the graph of f at some point. This just means there is a horizontal tangent line and $\frac{dy}{dx} = 0$ somewhere. Where ever $\frac{6}{y-2} = 5x^2$, there is a horizontal tangent line for f(x).

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Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

$5C_2$

Work for problem 5(c)

$f(-1) = -4$

$f'(-1) = 6$

$f''(-1) = 9$

$$P_2(x) = -4 + 6(x-1) + \frac{9(x-1)^2}{2}$$

Work for problem 5(d)

$\Delta x = .5$

x	y	$\Delta y = \frac{dy}{dx}\Delta x$	$\Delta y + y$
-1	-4	$= 6(.5)$	$3 + -4$
$-.5$	-1	$= \frac{9}{4}(.5)$	$\frac{9}{8} - 1$
0	$\frac{1}{8}$		

$f(0) \approx \frac{1}{8}$

$= 5x^2 - \frac{6}{y-2}$

$= \frac{5}{4} - \frac{-6}{-3}$

$= \frac{5}{4} + 2$

$= \frac{5+4}{4}$

$= \frac{9}{4} \times \frac{1}{2} \Rightarrow \frac{9}{8}$

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Question 5

Overview

This problem presented students with a differential equation and asked questions about a particular solution satisfying a given initial condition. In part (a) they needed to evaluate the first and second derivatives at the initial condition, using implicit differentiation for the latter computation. In part (b) students could observe that $\frac{dy}{dx} > 0$ when $y = 0$ to help decide if it was possible for the *x*-axis to be tangent to the graph of the particular solution at some point. Part (c) asked for the second-degree Taylor polynomial at $x = -1$, which the students could compute using the initial condition and the results from part (a). In part (d) students needed to use Euler's method with two steps of equal size to approximate the value of the particular solution at $x = 0$. Because the differential equation was not separable, students were not expected to solve the equation in order to answer these questions about the behavior of the particular solution. All questions could be answered by working directly with the differential equation.

Sample: 5A
Score: 9

The student earned all 9 points.

Sample: 5B
Score: 6

This student earned 6 points: 3 points in part (a), 1 point in part (c), and 2 points in part (d). The work in part (a) is correct. In part (b) the student fails to let $y = 0$. Thus the student did not earn the first point and was unable to successfully answer and explain for the second point. In part (c) the student does not show the quadratic centered at $x = -1$; however, the coefficients are correct. Thus the student earned the second, but not the first, point. The work in part (d) is correct.

Sample: 5C
Score: 4

This student earned 4 points: 2 points in part (a), 1 point in part (c), and 1 point in part (d). In part (a) the student earned the first 2 points but did not earn the last point due to a computational error. In part (b) the student never considers $y = 0$ so could not earn the first point and was therefore ineligible for the last point. In part (c) the quadratic polynomial is centered at $x = 1$ instead of at $x = -1$, so the student did not earn the first point. The coefficients are correct and/or consistent with part (a) and the second point was earned. In part (d) the student uses two steps of Euler's method, which earned the first point. A computational error prevents the student from getting the correct approximation to $f(0)$.

Question 6

The function f is defined by the power series

$$f(x) = -\frac{x}{2} + \frac{2x^2}{3} - \frac{3x^3}{4} + \cdots + \frac{(-1)^n nx^n}{n+1} + \cdots$$

for all real numbers x for which the series converges. The function g is defined by the power series

$$g(x) = 1 - \frac{x}{2!} + \frac{x^2}{4!} - \frac{x^3}{6!} + \cdots + \frac{(-1)^n x^n}{(2n)!} + \cdots$$

for all real numbers x for which the series converges.

(a) Find the interval of convergence of the power series for f. Justify your answer.

(b) The graph of $y = f(x) - g(x)$ passes through the point $(0, -1)$. Find $y'(0)$ and $y''(0)$. Determine whether y has a relative minimum, a relative maximum, or neither at $x = 0$. Give a reason for your answer.

(a) $$\left| \frac{(-1)^{n+1}(n+1)x^{n+1}}{n+2} \cdot \frac{n+1}{(-1)^n nx^n} \right| = \frac{(n+1)^2}{(n+2)(n)} \cdot |x|$$

$$\lim_{n\to\infty} \frac{(n+1)^2}{(n+2)(n)} \cdot |x| = |x|$$

The series converges when $-1 < x < 1$.

When $x = 1$, the series is $-\frac{1}{2} + \frac{2}{3} - \frac{3}{4} + \cdots$

This series does not converge, because the limit of the individual terms is not zero.

When $x = -1$, the series is $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots$

This series does not converge, because the limit of the individual terms is not zero.

Thus, the interval of convergence is $-1 < x < 1$.

$5 : \begin{cases} 1 : \text{sets up ratio} \\ 1 : \text{computes limit of ratio} \\ 1 : \text{identifies radius of convergence} \\ 1 : \text{considers both endpoints} \\ 1 : \text{analysis/conclusion for both endpoints} \end{cases}$

(b) $f'(x) = -\frac{1}{2} + \frac{4}{3}x - \frac{9}{4}x^2 + \cdots$ and $f'(0) = -\frac{1}{2}$.

$g'(x) = -\frac{1}{2!} + \frac{2}{4!}x - \frac{3}{6!}x^2 + \cdots$ and $g'(0) = -\frac{1}{2}$.

$y'(0) = f'(0) - g'(0) = 0$

$f''(0) = \frac{4}{3}$ and $g''(0) = \frac{2}{4!} = \frac{1}{12}$.

Thus, $y''(0) = \frac{4}{3} - \frac{1}{12} > 0$.

Since $y'(0) = 0$ and $y''(0) > 0$, y has a relative minimum at $x = 0$.

$4 : \begin{cases} 1 : y'(0) \\ 1 : y''(0) \\ 1 : \text{conclusion} \\ 1 : \text{reasoning} \end{cases}$

Work for problem 6(a)

$$\lim_{n\to\infty}\left|\frac{\dfrac{(-1)^{n+1}(n+1)x^{n+1}}{n+2}}{\dfrac{(-1)^n\cdot n\cdot x^n}{n+1}}\right| \Rightarrow \lim_{n\to\infty}\frac{\cancel{(-1)^{n+1}}\cdot(n+1)\cdot x^{n+1}\cdot(n+1)}{\cancel{(-1)^n}\cdot n\cdot x^n\cdot(n+2)}$$

$$= \lim_{n\to\infty}|x|$$

$$\Rightarrow R = 1$$

$$\Rightarrow \boxed{(-1\;,\;1)}$$

$(-1)^{2n}\dfrac{n}{n+1}$ diverges due to the n^{th} term test

$$\lim_{n\to\infty}\frac{(-1)^n\cdot n\cdot(-1)^n}{n+1}\neq 0$$

diverges due to the n^{th} term test

$$\lim_{n\to\infty}\frac{(-1)^n\cdot n\cdot 1^n}{n+1}\neq 0$$

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Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(b)

$g(x) = \cos x$

$$y' = \frac{(-1)^n \cdot n \cdot x^{n-1}}{(n+1)(n-1)} - \frac{(-1)^n \cdot x^{n-1}}{(2n)!\,(n+1)} \therefore y'(0) = 0$$

$$f'(x) = -1/2 + \tfrac{4}{3}x \ldots \quad \therefore f'(0) = -1/2$$

$$g'(x) = -1/2! + \tfrac{2}{4!}x \ldots \quad g'(0) = -1/2 \quad \therefore y'(0) = \boxed{0}$$

$$f''(x) = 4/3$$

$$g''(x) = 1/12 \quad \therefore y''(0) = 4/3 - 1/12 = \frac{15}{12} = \boxed{\frac{5}{4}}$$

has a relative min at $x=0$ because the derivative shows there is a critical point at $x=0$ and the 2nd derivative shows a positive concavity meaning the function values are decreasing up till $x=0$ and increasing after therefore showing a relative minimum

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STOP

END OF EXAM

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Work for problem 6(a)

$$\frac{(n+1)x^{\cancel{(n+1)}}}{(n+1)+1} \cdot \frac{n+1}{n\cancel{x^n}} = \frac{(n^2+2n+1)x}{(n^2+n)} = 3x$$

$-1 \leq 3x \leq 1$

$$\boxed{-\frac{1}{3} \leq x \leq \frac{1}{3}}$$

$\frac{(-1)^n n(-\frac{1}{3})^n}{n+1} \to$ diverges

$\frac{(-1)^n n(\frac{1}{3})^n}{n+1} \to$ converges

alternating

* when $-\frac{1}{3} < x \leq \frac{1}{3}$, the power series for f converges.

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Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(b)

$y' = f'(x) - g'(x)$

$y'(0) = -\frac{1}{2} + \frac{1}{2} = \boxed{0}$

$y'' = f''(x) - g''(x)$

$y''(0) = \frac{4}{3} - \frac{1}{12} =$

$\frac{16}{12} - \frac{1}{12} = \boxed{\frac{15}{12}}$

$f'(x) = -\frac{1}{2} + \frac{4x}{3} \ldots$

$f''(x) = \frac{4}{3}$

$g'(x) = -\frac{1}{2} + \frac{1}{12}x$

$g''(x) = \frac{1}{12}$

y has a relative minimum at x=0 because the derivative of y at x=0 is 0 and the second derivative of y at x=0 is >0, meaning the graph of y is concave up at this point.

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NO CALCULATOR ALLOWED

Work for problem 6(a)

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n n x^n}{n+1}$$

$$L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$L = \lim_{n \to \infty} \left| \frac{(-1)^{n+1}(n+1)x^{n+1}}{n+2} \cdot \frac{n+1}{(-1)^n n x^n} \right|$$

$$L = \lim_{n \to \infty} \left| (-1)x \cdot \frac{(n+1)^2}{(n+2)(n)} \right|$$

$$L = x$$

$$L < 1$$

$$x < 1$$

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Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(b)

$y = f(x) - g(x)$

$y'(x) = \left[-\frac{1}{2} + \frac{4x}{3} + \dots\right] - \left[-\frac{1}{2} + \frac{2x}{4!} + \dots\right]$

$y'(0) = -\frac{1}{2} - \left(-\frac{1}{2}\right)$

$y'(0) = 0$

$y''(x) = \left[\frac{4}{3} - \frac{18}{4}x + \dots\right] - \left[\frac{2}{24} - \frac{6x}{6!} + \dots\right]$

$y''(0) = \frac{4}{3} - \frac{1}{12} = \boxed{\frac{15}{12}}$

x < — + > 0

relative minimum

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Question 6

Overview

This problem dealt with power series. Students were given the power series expansions of two functions, f and g. In part (a) they were asked to find the interval of convergence of the power series for f. Part (b) dealt with the graph of $y = f(x) - g(x)$. Students had to know how to read or compute the values of the first and second derivatives of y at $x = 0$ from the series for f and g. They then needed to use this information to describe the nature of the critical point of y at $x = 0$.

Sample: 6A
Score: 9

The student earned all 9 points. In part (b) the student restarts the problem on the third line and earned all points.

Sample: 6B
Score: 6

The student earned 6 points: 2 points in part (a) and 4 points in part (b). In part (a) the student sets up an incorrect Ratio Test and does not compute a limit. The student earned the 2 points by stating the correct interval of convergence for the ratio and testing the endpoints. In part (b) the student calculates $y'(0)$ and $y''(0)$ and arrives at the correct conclusion with good reasoning.

Sample: 6C
Score: 4

The student earned 4 points: 1 point in part (a) and 3 points in part (b). In part (a) the student sets up the ratio correctly but does not identify the correct interval of convergence. In part (b) the student correctly calculates $y'(0)$ and $y''(0)$ and arrives at a correct conclusion. However, the student does not include a reason for the answer.